A Blind Antenna Selection Scheme for Single-cell Uplink Massive MIMO

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***Abstract*—This paper considers the uplink of a single-cell large-scale multiple-input multiple output (MIMO) system in which m mono-antenna users communicate with a base station (BS) outfitted by n antennas. We assume that the number of antennas at the BS and that of users take large values, as envisioned by large-scale MIMO systems. This allows for high spectral efficiency gains but obviously comes at the cost of higher complexity, a fact that becomes all the more critical as the number of antennas grows large. To solve this issue is to choose a subset of the available n antennas. The subset must be carefully chosen to achieve the best performance. However, finding the optimal subset of antennas is usually a difficult task, requiring one to solve a high dimensional combinatorial optimization problem. In this paper, we approach this problem in two ways. The first one consists in solving a convex relaxation of the problem using standard convex optimization tools. The second technique solves the problem using a greedy approach. The main advantages of the greedy approach lies in its wider scope, in that, unlike the first approach, it can be applied irrespective of the considered performance criterion. As an outcome of this feature, we show that the greedy approach can be applied even when only the channel statistics are available at the BS, which provides blind way to perform antenna selection.**

***Index Terms*—Massive MIMO, antenna selection, convex optimization, greedy algorithm, blind selection, random matrix theory.**

I. Introduction

Massive MIMO, also known as large-scale antenna systems, are considered as a promising technology for future wireless standards [1]. By deploying large antenna arrays at the base station (BS), massive MIMO systems are capable of achieving remarkable performance improvement in terms of capacity, radiated energy efficiency and link reliability [1]–[3]. As a rule of thumb, in the realm of massive MIMO, hundreds of antennas at the BS are used to serve several tens of users. However, as mentioned in [4], it can be very expensive to deploy radio-frequency (RF) elements for all the antennas. Moreover, as far as detection is considered, using all available antennas might not be practically reasonable, as it fails to achieve a good balance between complexity and performance. One solution addressing this issue is to activate only a portion of the total number of antennas. This naturally calls for antenna selection as a potential approach to reduce the detection complexity and cost while maintaining the system performance at a certain level.

The concept of antenna selection has been extensively studied within the field of signal processing. For example, a similar problem known as sensor selection has been motivated by several applications, ranging from robotics, target tracking to wireless networks. For more details, we refer the reader to the work in [5] and the references therein. Similarly, antenna selection has already been advocated as an efficient solution to reduce the number of RF chains [6,7] in conventional MIMO systems, leading to a significant reduction in complexity and costs while preserving most of the potential of full MIMO systems. Basically, k antennas out of n available antennas are selected in order to minimize/maximize a certain selection metric. The selection metric can be for instance, the maximization of the signal-to-noise ratio (SNR) at the base station, or the maximization of the channel capacity/sum-rate or the system diversity, etc [4].

In this work, we consider the application of antenna selection to massive MIMO systems. In particular, we consider the uplink of a single cell multi-user MIMO system in which the BS equipped with n antennas receives signals from m single-antenna users. Instead of using all the available antennas, we assume that the BS selects k out of the n available antennas. As per the antenna selection procedure, these k antennas must be chosen in order to minimize a certain metric. Of particular interest in our work, is the case where the BS performs zero-forcing detection. It is thus sensible to select the k antennas that minimize the *mean square error* (MSE) at the receiver. This can be formulated as a combinatorial optimization problem with n parameters and comprising n Boolean constraints. One naive way to solve this problem is to resort to an exhaustive search that would require $\binom{n}{k}$ computations. This solution, however, could not obviously be implemented as it goes against the initial target of reducing the computational complexity. An alternative to this solution, which can find its roots in the work of [5], is based on the use of convex relaxation techniques. This method merely consists in solving a convex-related problem, which is obtained using convex constraints instead of the original ones. However, even though the complexity is not as prohibitive as in the exhaustive search, it is still high, being approximately in the order of $\mathcal{O}\left(n^3\right)$. To deal with this issue, a greedy algorithm that attempts to approximate the optimal solution can be implemented [1]. As will be shown in the course of the paper, the greedy algorithm has two main advantages. First, it allows a considerable reduction in complexity requiring roughly $\mathcal{O}\left(n^2\right)$ operations. Second, it can be applied to a wide range of metrics of interest. This implies in particular that we can consider the minimization of an average metric instead of the mean square error. In particular, for sake of illustration, we use in this paper the asymptotic equivalent of the MSE based on some results from random matrix theory (RMT). In doing so, only the statistics of the channel have to be known. This not only alleviates the requirement of acquiring the channel state information (CSI) at the BS, but also allows a further reduction in performance as it suggests to perform antenna selection at the rate of the change of the channel statistics. Interestingly, the numerical results show that the degradation in performances with respect to the channel-aware greedy algorithm is small for all practical values of k.

The remainder of the paper is organized as follows. In section II, we present the system model and the general concept of antenna selection in massive MIMO. In section III, we introduce the different proposed schemes to perform antenna selection. In section IV, we provide some numerical results illustrating the efficiency of the proposed selection algorithms. We then conclude the paper in section V.

Notations: Throughout the paper, we use the following notations: Matrices are denoted by bold capital letters ($\mathbf{I}_n$ is the identity matrix of size n) and vectors are denoted by lower case bold letters. For a given matrix $\mathbf{A}$, we refer by $[\mathbf{A}]_{i,j}$ its (i,j)th entry, and use $\mathbf{A}^T$ and $\mathbf{A}^H$ to denote its transpose and Hermitian respectively. We respectively denote by $\|.\|$ and $\operatorname{tr}(.)$, the spectral norm and the trace of a matrix. Finally, we denote by $\operatorname{diag}(\mathbf{a})$, the diagonal matrix with diagonal elements the entries of $\mathbf{a}$.

II. SYSTEM DESCRIPTION

We consider the uplink of a single-cell MU-MIMO system in which m single-antenna users are served by a single base station (BS) equipped with n antenna with $m \leq n$, as sketched in Figure 1. We assume that the users' signals are perfectly synchronized in time and frequency. Thus, the received signal vector at the BS can be expressed as:

$$\mathbf{y} = \sqrt{\rho}\mathbf{H}\mathbf{x} + \mathbf{e}, \tag{1}$$

where $\mathbf{y} \in \mathbb{C}^{n\times 1}$ is the received vector at the BS, ρ is the average transmit power per user and $\mathbf{x} \in \mathbb{C}^{m\times 1}$ is the data vector. The matrix $\mathbf{H} = \{h_{i,j}\} \in \mathbb{C}^{n\times m}$ denotes the narrow-band uplink channel matrix where $h_{i,j}$ is the channel coefficient between the j-th user and the i-th BS's antenna. Moreover, we assume that the random channel $\mathbf{H}$ exhibits the one-sided Kronecker model given by

$$\mathbf{H} = \mathbf{\Theta}_R^{\frac{1}{2}}\mathbf{G}, \tag{2}$$

where $\mathbf{G} \in \mathbb{C}^{n\times m}$ is a matrix with $i.i.d$ circularly symmetric zero mean unit-variance complex Gaussian entries and $\mathbf{\Theta}_R$ models the receive correlation matrix, whose elements represent the correlation between the antennas of the BS. In particular,

$$\mathbf{\Theta}_R \triangleq \mathbb{E}\left[\mathbf{H}\mathbf{H}^H\right].$$

Finally, $\mathbf{e} \in \mathbb{C}^{n\times 1}$ denotes the noise vector at the BS with $i.i.d$ circularly symmetric zero mean unit-variance complex Gaussian entries, i.e., $\mathbf{e} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_n)$.

At the receiver side, the BS estimates the transmitted vector $\mathbf{x}$ on the basis of the observation vector $\mathbf{y}$. Several detection procedures can be used, among which are the optimal maximum likelihood (ML) detector and the least-Square estimator. The latter achieves a good balance between complexity and performance. In communication parlance, it is referred to as zero-forcing detection and is given by

$$\widehat{\mathbf{x}} = \frac{1}{\sqrt{\rho}}\mathbf{H}^{\dagger}\mathbf{y},$$

where $\mathbf{H}^{\dagger} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H$ is the pseudo-inverse of $\mathbf{H}$. The performance of the ZF detector for a given channel realization is assessed using the *mean square error* (MSE) which is given by

$$\begin{aligned} \text{MSE} &\overset{(a)}{=} \mathbb{E}\left[\|\widehat{\mathbf{x}} - \mathbf{x}\|^2\right] \\ &= \operatorname{tr}[\mathbf{\Sigma}] \\ &= \frac{1}{\rho}\operatorname{tr}\left[\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\right] \\ &= \frac{1}{\rho}\operatorname{tr}\left[\left(\mathbf{G}^H\mathbf{\Theta}_R\mathbf{G}\right)^{-1}\right], \end{aligned} \tag{3}$$

where the expectation in (a) is taken over all the noise realizations and $\mathbf{\Sigma} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}$ denotes the estimation error covariance matrix.

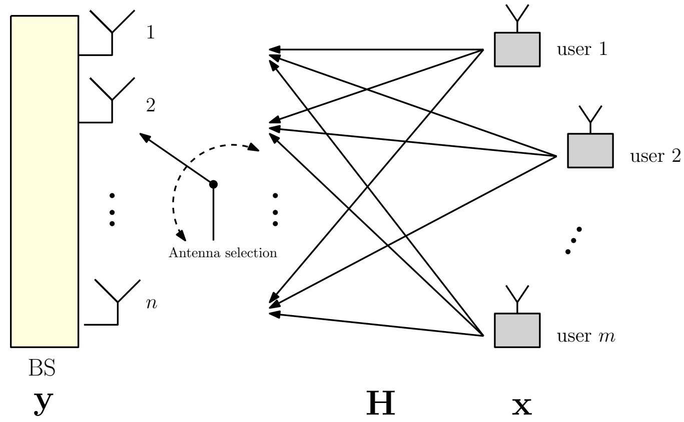

Figure 1: System model of a MU-MIMO system in the uplink with a BS equipped with n antennas and serving m single-antenna users.

[1]The greedy approach has been proposed in many scenarios namely user scheduling in multiuser networks.

A. Antenna Selection

Based on the available CSI at the BS, the goal of antenna selection is to select the "best" k antennas out of n antennas that minimizes a given performance metric, where $m \leq k \leq n$. Obviously, reducing the number of antennas to k is expected to cause a performance loss. However, there are many scenarios, such as those encountered in massive MIMO systems, where computational complexity can be of a major concern. In such circumstances, achieving a substantial reduction in complexity at the cost of an acceptable degradation in performances is viewed as a good option. In this paper, we propose to reduce complexity by performing antenna selection, where we keep the k antennas that yield the least value of MSE. More specifically, for a given set of indexes $\mathcal{S}$ ($|\mathcal{S}| = k$), containing the index of antennas to be selected , we define the selection matrix $\mathbf{S} \in \mathbb{R}^{k\times n}$ as the matrix that permits to extract k measurements from $\mathbf{H}$ corresponding to $\mathcal{S}$. This can be mathematically written as

$$\mathbf{y}_{\mathcal{S}} = \mathbf{S}\mathbf{y}, \tag{4}$$

where $\mathbf{S}$ is a $k \times n$ matrix defined by

$$[\mathbf{S}]_{i,j} = \begin{cases} 1 & j = \mathcal{S}[i] \\ 0 & \text{otherwise} \end{cases}, i = 1, \cdots, k. \tag{5}$$

In other words, $\mathbf{S}$ is a matrix which exalts one non zero entry in each row located at the columns indexed by the set $\mathcal{S}$. Using the structure of $\mathbf{S}$ in (5), we have the following properties

- $\mathbf{S}\mathbf{S}^T = \mathbf{I}_k$.
- $\mathbf{S}^T\mathbf{S} = \text{diag}(\mathbf{s})$.

where $\mathbf{s} = \{s_i\}_{i=1,\cdots,n}$ is a $n-$dimensional vector with entries equal to 1 at the locations given by $\mathcal{S}$ and zero elsewhere. Upon applying the operator defined by $\mathbf{S}$, the resultant error covariance matrix, which we denote by $\mathbf{\Sigma}_{\mathcal{S}}$, writes as

$$\begin{aligned} \mathbf{\Sigma}_{\mathcal{S}} &= \left(\mathbf{H}^H\mathbf{S}^T\mathbf{S}\mathbf{H}\right)^{-1} \\ &= \left(\mathbf{G}^H\mathbf{\Theta}_R^{\frac{1}{2}}\mathbf{S}^T\mathbf{S}\mathbf{\Theta}_R^{\frac{1}{2}}\mathbf{G}\right)^{-1} \\ &= \left(\mathbf{G}^H\mathbf{\Theta}_R^{\frac{1}{2}}\text{diag}(\mathbf{s})\,\mathbf{\Theta}_R^{\frac{1}{2}}\mathbf{G}\right)^{-1}. \end{aligned} \tag{6}$$

As stated earlier, the optimal set of measurements denoted by $\mathcal{S}^*$ is chosen to minimize the MSE. Mathematically speaking, the selection problem can be formulated as follows

$$\begin{aligned} \mathbf{s}^* = \underset{\mathbf{s}}{\text{argmin}} \quad & \text{tr}\left[\left(\mathbf{G}^H\mathbf{\Theta}_R^{\frac{1}{2}}\text{diag}(\mathbf{s})\,\mathbf{\Theta}_R^{\frac{1}{2}}\mathbf{G}\right)^{-1}\right] \\ \text{s.t.} \quad & \mathbf{1}^T\mathbf{s} = k \\ & s_i \in \{0,1\}, i = 1, \cdots, n. \end{aligned} \tag{7}$$

Obviously, finding the optimal solution of the problem in (7) might be too computationally cumbersome, as it involves Boolean constraints. To solve this issue, we provide in the next section two alternatives that provide suboptimal solutions to the problem in (7) with a considerable reduced complexity. The first algorithm is in the spirit of the work of [5] and merely relies on using convex relaxation techniques allowing to substitute the constraints by convex ones. The second algorithm solves the problem using a greedy approach and presents even lower complexity compared to the first one.

III. Sub-optimal Antenna Selection

In this section, we propose two different sub-optimal solutions to the problem in (7). The first one, which is inspired by the work of [5], merely consists in relaxing the Boolean constraints of (7) into convex ones. This provides a related convex problem whose solution corresponds to a suboptimal design. The second approach is to use a greedy algorithm that attempts to provide a reasonable sub-optimal solution to (7). The main advantages of the latter approach are twofold. First, it presents a lower complexity. Second, it can be applied to any metric of interest. Particularly, when applied to average metrics depending only on the channel statistics, the greedy algorithm provides a blind way to select the antennas, avoiding thus for the BS the need to acquire CSI.

A. CSI aware antenna selection approaches

1) Antenna Selection via Convex Optimization: This approach is based on solving a convex related problem to the one in (7). Since the objective function is convex for $s_i \geq 0, i = 1, \cdots, n$, this problem is obtained by simply replacing the boolean constraints $s_i \in \{0,1\}$ with the convex constraints $s_i \in [0,1]$. In doing so, we obtain the following optimization problem

$$\begin{aligned} \mathbf{s}_0 = \underset{\mathbf{s}}{\text{argmin}} \quad & \text{tr}\left[\left(\mathbf{G}^H\mathbf{\Theta}_R^{\frac{1}{2}}\text{diag}(\mathbf{s})\,\mathbf{\Theta}_R^{\frac{1}{2}}\mathbf{G}\right)^{-1}\right] \\ \text{s.t.} \quad & \mathbf{1}^T\mathbf{s} = k \\ & 0 \leq s_i \leq 1, i = 1, \cdots, n. \end{aligned} \tag{8}$$

It is worth mentioning that the output of the optimization in (8) yields a lower value than the minimum MSE, and as such can be viewed as a global lower bound on the performance. Moreover, the optimal vector $\mathbf{s}_0$ can contain real values not necessarily zeros or ones. In order to obtain the index of the selected antennas, one should order the elements of $\mathbf{s}_0$ and then assign ones to the k-th greatest values and zeros to the remaining entries.

2) Greedy Approach: Greedy algorithms have been widely applied in the framework of wireless communication, particularly in scheduling where the objective is to select the set of users that maximizes a certain utility function [8]. The use of greedy algorithms for antenna selection is, however, less common in the context of antenna selection and measurement selection in general. In order to stress the wide scope of applicability of the proposed greedy algorithm, we consider here the problem of selecting the index of antenna elements that minimize a pre-defined metric $f(\mathcal{H}, \mathcal{S})$, where $\mathcal{H}$ is some information about the channel $\mathbf{H}$ [2] and $\mathcal{S}$ is a set of k indexes from $\{1, \cdots n\}$. The principle of the proposed greedy algorithm is as follows. First, we start by choosing an initial candidate set $\mathcal{S}$ obtained by randomly selecting a pattern (set of antenna indexes) of size k. Then, we select from the set of the remaining indexes ($\overline{\mathcal{S}} = \{1, \cdots, n\} \backslash \mathcal{S}$), the first value that, when replaced with one of the indexes in $\mathcal{S}$ leads to a reduction in $f(\mathcal{H}, \mathcal{S})$. When this occurs, $\mathcal{S}$ is updated by replacing the index that presents the largest reduction

[2] $\mathcal{H}$ could be for example the channel statistics or the full channel itself.

Algorithm 1 Greedy Approach for Antenna Selection

Initialize $\mathcal{S} = randsample(n, k)$ ▷ **randomly generate a pattern of size** k
Compute metric$^{*} = f(\mathcal{H}, \mathcal{S})$
for $i = 1 \to \#iterations$ **do**
    $\overline{\mathcal{S}} = \{1, \cdots, n\} \setminus \mathcal{S}$
    $j \leftarrow 1$
    while $j \leq n - k$ **do**
        $p \leftarrow \overline{\mathcal{S}}[j]$
        $\mathcal{I} \leftarrow \mathcal{S}$
        table $\leftarrow$ **zeros** $(k, 1)$
        for $l = 1 \to k$ **do**
            $\mathcal{I}[l] \leftarrow p$
            table $[l] \leftarrow f(\mathcal{H}, \mathcal{I})$
            $\mathcal{I} \leftarrow \mathcal{S}$
        end for
        if $\min$ (**table**) $<$ **metric*** **then**
            metric$^{*} \leftarrow \min$ (**table**)
            $\mathcal{S}[\arg\min$ (**table**)$] \leftarrow p$
        end if
    end while
end for

in $f(\mathcal{H}, \mathcal{S})$. This procedure is repeated for a predetermined number of iterations. The corresponding algorithm is detailed in Algorithm 1. It can be applied for any metric $f(\mathcal{H}, \mathcal{S})$ but the focus in this paper is on the use of MSE.

Proposition 1. *The greedy algorithm described by the steps of Algorithm 1 is guaranteed to converge.*

Proof: By construction of Algorithm 1, the computation metric $f(\mathcal{H}, \mathcal{S})$ decreases from one iteration to the next. Since the performance is bounded by the optimal performance achieved by exhaustive search, then $f(\mathcal{H}, \mathcal{S})$ is decreasing and bounded from below which gives the claim of Proposition 1. ∎

B. Blind Antenna Selection

1) Motivation: In many applications, the channel matrix $\mathbf{H}$ is not constant, i.e. it can randomly change from time to time. For example, $\mathbf{H}$ follows a block fading model in which the channel matrix is taken to be constant over a given duration known as block and change independently from one block to the other. That being said, the antenna selection process has to track the variation of the channel matrix and thus antenna selection has to be performed at every realization of $\mathbf{H}$. Assuming, we need to perform antenna selection over a period of N blocks, the complexity then scales by a factor of N yielding a complexity of $N \times \mathcal{O}(n^3)$ computations in the case of convex optimization. This becomes more tedious especially when the number of blocks is large. Moreover, in many situations, the channel matrix $\mathbf{H}$ is unknown or can not be estimated, thus, the only way to perform antenna selection is via a approach. This constitutes the main motivations to develop selection algorithms that can perform antenna selection without the need to track the variation in $\mathbf{H}$.

2) Main idea: The idea is to consider the use of an approximate of the MSE. This approximate is obtained from random matrix theory results showing that the average MSE converge to a deterministic quantity as both dimensions n and m grow simultaneously large without bounds while their ratio $\frac{m}{k} \to c \in (0, 1)$. In particular, we have the following result

Lemma 1. *[9, Theorem 2] Let $\mathbf{\Lambda}(\mathbf{s}) = \mathbf{\Theta}_R^{\frac{1}{2}} diag(\mathbf{s}) \mathbf{\Theta}_R^{\frac{1}{2}}$ with non zero eigenvalues $\lambda_i(\mathbf{s})$, $i = 1, \cdots, k$. Then, as m and n go simultaneously large without bound*

$$\frac{1}{m}\mathbb{E}\, tr\left[\left(\mathbf{G}^*\mathbf{\Theta}_R^{\frac{1}{2}} diag(\mathbf{s}) \mathbf{\Theta}_R^{\frac{1}{2}}\mathbf{G}\right)^{-1}\right] - \overline{m}_{\mathbf{\Lambda}(\mathbf{s})} \xrightarrow[\frac{m}{n} \to c]{a.s.} 0,$$

where $\overline{m}_{\mathbf{\Lambda}(\mathbf{s})}$ is the unique solution to the following fixed-point equation

$$\overline{m}_{\mathbf{\Lambda}(\mathbf{s})} = \frac{m}{\sum_{i=1}^{n} \frac{\lambda_i(\mathbf{s})}{1+\lambda_i(\mathbf{s})\overline{m}_{\mathbf{\Lambda}(\mathbf{s})}}}. \tag{9}$$

The quantity $\overline{m}_{\mathbf{\Lambda}(\mathbf{s})}$ is well-known as the asymptotic inverse moment of the Gram matrix $\mathbf{G}^*\mathbf{\Theta}_R^{\frac{1}{2}} \text{diag}(\mathbf{s}) \mathbf{\Theta}_R^{\frac{1}{2}}\mathbf{G}$ with one-side correlation given by $\text{diag}(\mathbf{s}) \mathbf{\Theta}_R$. For more information about inverse moments of one-sided correlated Gram matrices, the readers are referred to our work in [9] and references therein. Using the asymptotic inverse moments derived in Lemma 1, we formulate the approximate antenna selection problem as follows

$$\begin{aligned} \mathbf{s}^* = \underset{\mathbf{s}}{\text{argmin}} \quad & \overline{m}_{\mathbf{\Lambda}(\mathbf{s})} \\ \text{s.t.} \quad & \mathbf{1}^T\mathbf{s} = k \\ & s_i \in \{0, 1\}, i = 1, \cdots, n. \end{aligned} \tag{10}$$

As shown in the formulation in (10), the objective function depend only on the large scale channel statistics and can be computed beforehand, or at least be updated at the rate of change of the channel statistics. Therefore, blind antenna selection can be directly obtained from the greedy algorithm described in Algorithm 1 by simply selecting an average metric $f(\mathcal{H}, \mathcal{S})$ depending solely on the channel statistics of $\mathbf{H}$ given by $\mathbf{\Theta}_R$.

IV. Selected Numerical Results

In this section, we present some numerical results in order to compare between the different proposed antenna selection approaches [3]. All the experiments are performed when the number of users m is set to 10, a total budget of antennas is $n = 80$ antennas and an SNR of $\rho = 20$ dB. Moreover, we consider the following spatial correlation model [10]

$$[\mathbf{\Theta}_R]_{i,j} = \exp\left(-0.05.d^2 (i-j)^2\right). \tag{11}$$

This models a broadside Gaussian power azimuth spectrum with 2° root-mean-square spread where d corresponds to the wavelength antenna separation. For the channel-aware implementation ($\mathcal{H} = \mathbf{H}$), we consider both approaches, the greedy and the one based on convex optimization, however,

[3]The algorithm based on an exhaustive search is omitted due to its huge complexity.

for the blind implementation ($\mathcal{H} = \mathbf{\Theta}_R$), we only consider the greedy approach. To begin with, we illustrate the accuracy of the asymptotic equivalent by representing the exact value of the objective function in (7) along with its asymptotic approximation for several values of k. As shown in Figure 2, the asymptotic equivalent provides an accurate approximation to the exact MSE especially for large k.

In Figure 3 , we plot the MSE performance achieved by the greedy algorithm for both cases (channel-aware and blind) as a function of the number of iterations. For both cases, the greedy algorithm requires a number of iterations, $K = 2$ to converge. This value of K will be implemented in all the next simulations for the greedy algorithm. In Figure 4, we show the performance of the proposed algorithms along with the random selection algorithm that randomly select a set of k antennas out of n. The main observation is that when the impact of correlation is small ($d = 2$), the proposed blind algorithm is not so advantageous as compared to the random selection algorithm since the rows of $\mathbf{H}$ are quasi-independent and so the blind approach is not of much use. As we increase the impact of correlation ($d \downarrow$), the blind approach has a significant gain compared to the random approach and gives slightly lower performance as compared to the algorithms that require full channel knowledge. The MSE in Figures 2 and 4 is averaged over $N = 100$ channel realizations, i.e.

$$\overline{\text{MSE}} = \frac{1}{N}\sum_{i=1}^{N} \text{MSE}\,(i)\,, \qquad (12)$$

where MSE (i) is the MSE for the ith realization. Note that, the blind greedy algorithm performs antenna selection only once as long as the statistics are unchanged and as such requires lower computational complexity as illustrated in Table I and Figure 5 . All in all, it appears that it presents in reality a better trade-off between complexity and performance.

Algorithm	Complexity
Convex Optimization	$N \times \mathcal{O}\left(n^3\right)$[5]
Greedy ($\mathcal{H} = \mathbf{H}$)	$K \times N \times \mathcal{O}\left(n^2\right)$
Greedy ($\mathcal{H} = \mathbf{\Theta}_R$)	$K \times \mathcal{O}\left(n^2\right)$

Table I: Computational complexity of the different proposed algorithms.

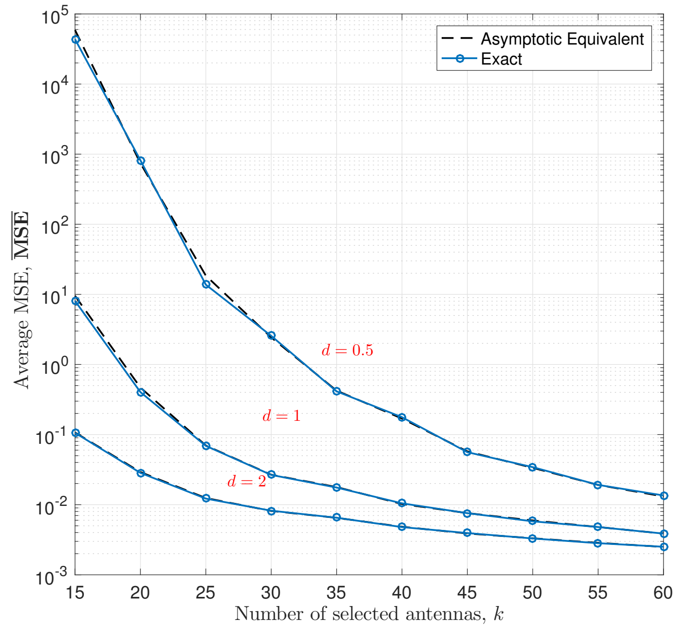

Figure 2: MSE performance: Exact (3) vs. Asymptotic equivalent solution to the fixed-point equation in (9) for different values of the antennas' separation, d.

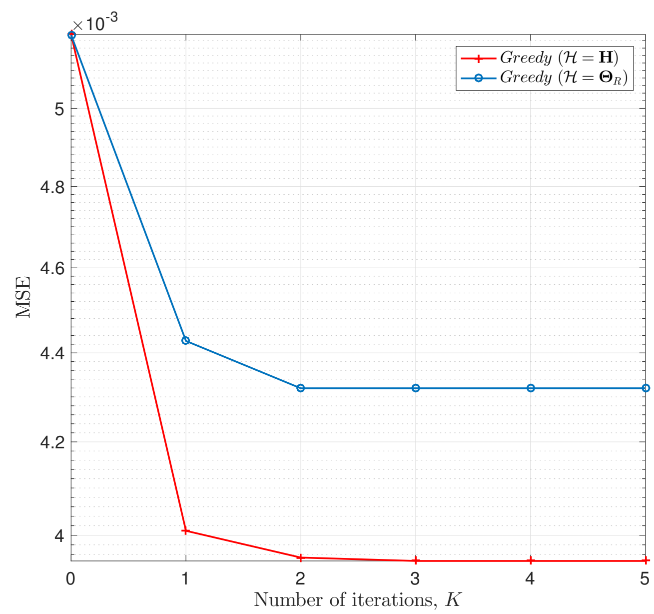

Figure 3: MSE performance of the greedy approach for both cases (channel-aware and blind) as function of the number of iterations, K ($k = 40$ selected antennas and $d = 1.5$).

V. Conclusion

This paper considered the use of antenna selection for massive MIMO systems to reduce the detection complexity at the BS. The selected antenna should be chosen such that a given performance metric is minimized. This in general yields a difficult combinatorial optimization problem. In this paper, we proposed two heuristic algorithms to approximately solve this problem. The first one relies on convex optimization tools and merely consists in solving a related-convex problem. The second one is based on a greedy algorithm. Interestingly, we showed that this algorithm can be also applied when only the channel statistics are available. Numerical results are presented in order to compare between all the different approaches in terms of performance and complexity.

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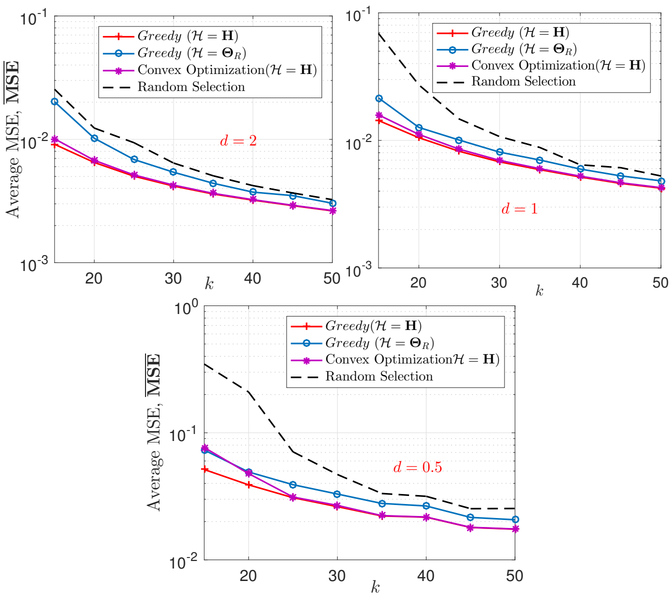

Figure 4: Average MSE (over $N = 100$ channel realizations) vs. the number of selected antennas, k for different values of the antennas' separation, d.

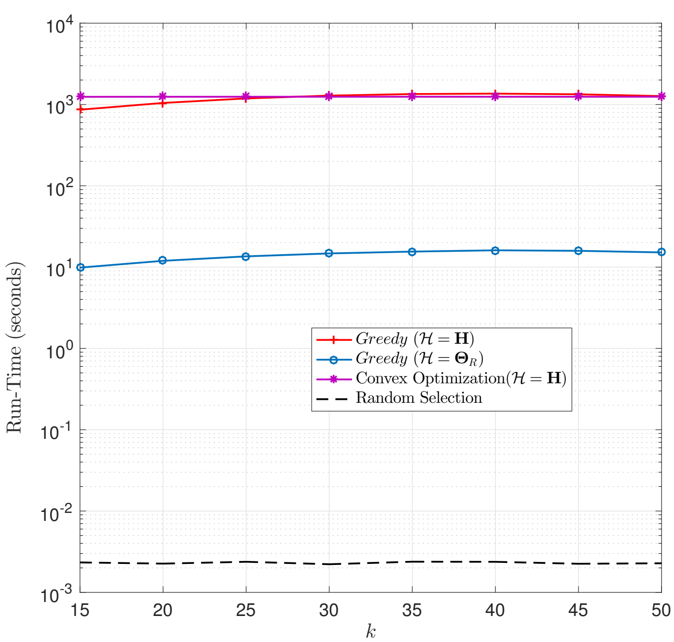

Figure 5: Run-time is seconds vs the number of selected antennas, k, for the considered selection algorithms.

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